



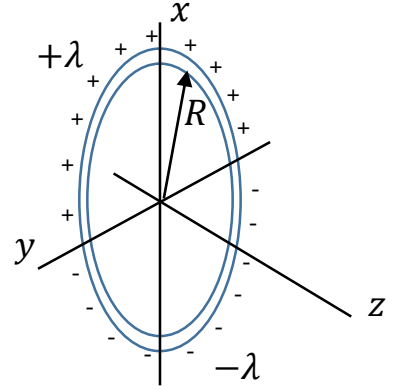
PHYS 102 – General Physics II Midterm Exam

Duration: 90 minutes

Saturday, 19 October 2024; 16:00

1. A thin non-conducting ring of radius R is placed on the xy -plane with its center at the origin. Half of the ring ($x > 0$) is charged positively with uniform charge density $+\lambda$, and the other half ($x < 0$) is charged negatively with uniform charge density $-\lambda$.

- (5 Pts.) what is the total charge in the upper half ($x > 0$) of the ring?
- (5 Pts.) What is the electric potential on a point on the z axis?
- (5 Pts.) What is the z -component of the electric field on the z -axis?
- (5 Pts.) In which direction is the electric field pointing at the same point?
- (15 Pts.) Find the magnitude of the electric field at the same point.



Solution: (a) $Q_t = \pi R \lambda$

(b) Take an infinitesimal piece of the top ring with charge dq . Potential on the z -axis is found as

$$V_+(z) = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{\sqrt{R^2 + z^2}} \rightarrow V_+(z) = \frac{R \lambda}{4\epsilon_0 \sqrt{R^2 + z^2}}$$

Similarly, for the bottom ring we have

$$V_-(z) = \frac{-R \lambda}{4\epsilon_0 \sqrt{R^2 + z^2}}$$

Therefore the net potential on the z -axis is $V(z) = V_+(z) + V_-(z) = 0$.

(c)

$$E_z = \frac{dV}{dz} = 0.$$

(d) Because of the symmetry of the charge distribution, electric field on the z -axis will only be in the $-\hat{i}$ direction.

(e) Consider the top ring only. $dq = \lambda R d\theta$, $\vec{r} = z \hat{k}$, $\vec{r}' = R \cos \theta \hat{i} + R \sin \theta \hat{j}$ and $|\vec{r} - \vec{r}'| = \sqrt{R^2 + z^2}$, where θ ($-\pi/2 < \theta < \pi/2$) is the angle measured from the x -axis in the clockwise direction.

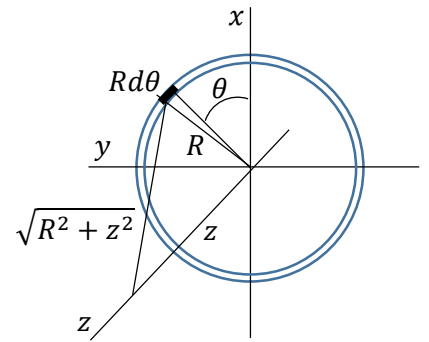
$$\vec{E}_+ = \frac{1}{4\pi\epsilon_0} \int_{-\pi/2}^{\pi/2} \frac{(\vec{r} - \vec{r}') dq}{|\vec{r} - \vec{r}'|^2} \rightarrow \vec{E}_+ = \frac{\lambda R}{4\pi\epsilon_0} \int_{-\pi/2}^{\pi/2} \frac{-R \cos \theta \hat{i} - R \sin \theta \hat{j} + z \hat{k}}{(R^2 + z^2)^{3/2}} d\theta$$

$$\vec{E}_+ = \frac{\lambda R}{4\pi\epsilon_0 (R^2 + z^2)^{3/2}} \left[-R \hat{i} \int_{-\pi/2}^{\pi/2} \cos \theta d\theta - R \hat{j} \int_{-\pi/2}^{\pi/2} \sin \theta d\theta + z \hat{k} \int_{-\pi/2}^{\pi/2} d\theta \right]$$

$$\vec{E}_+ = \frac{-2\lambda R^2 \hat{i}}{4\pi\epsilon_0 (R^2 + z^2)^{3/2}} + \frac{\lambda R z \pi \hat{k}}{4\pi\epsilon_0 (R^2 + z^2)^{3/2}}$$

Similarly, the electric field produced by the bottom ring is found as

$$\vec{E}_- = \frac{-2\lambda R^2 \hat{i}}{4\pi\epsilon_0 (R^2 + z^2)^{3/2}} - \frac{\lambda R z \pi \hat{k}}{4\pi\epsilon_0 (R^2 + z^2)^{3/2}} \rightarrow \vec{E} = \vec{E}_+ + \vec{E}_- = \frac{-\lambda R^2 \hat{i}}{\pi\epsilon_0 (R^2 + z^2)^{3/2}}$$



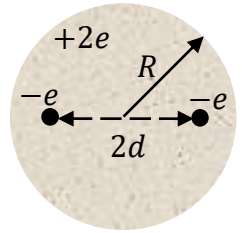
2. According to the (now discredited) Thompson atomic model, an atom is composed of a uniformly charged positive sphere, and negatively charged point particle electrons moving freely inside this sphere. According to this model the Helium atom would have two electrons, both with charge $-e$ inside a sphere of uniform charge density. This sphere will have radius R , and a total charge of $+2e$. In the equilibrium configuration, the two electrons will be on a line passing through the center of the sphere, separated by a distance of $2d$, as shown.

a) (10 Pts.) Find the magnitude of the force the two electrons apply to each other.

b) (13 Pts.) Find d so that the total force on the electrons is zero. (Express d in terms of R , e , and universal constants.)

c) (6 Pts.) What is the total electric flux through the outer surface of the sphere?

d) (6 Pts.) Find the magnitude of the electric field on the equatorial circle of the sphere equidistant from both electrons.



Solution: (a)

$$|\vec{F}_e| = \frac{1}{4\pi\epsilon_0} \frac{e^2}{4d^2} \rightarrow |\vec{F}_e| = \frac{e^2}{16\pi\epsilon_0 d^2}.$$

(b) To find the electric field inside the atom created by the positive charge distribution we use Gauss's law applied to a concentric sphere of radius $r < R$.

$$Q_{enc} = \rho V = \left(\frac{2e}{\frac{4}{3}\pi R^3} \right) \left(\frac{4}{3}\pi r^3 \right) \rightarrow Q_{enc} = \frac{2e r^3}{R^3}$$

$$\int \vec{E}_+ \cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0} \rightarrow E_+(4\pi r^2) = \frac{2e r^3}{R^3 \epsilon_0} \rightarrow E_+(r) = \frac{e r}{2\pi\epsilon_0 R^3}, \quad r < R.$$

The field is in the radial direction so the force $\vec{F}_+$ on the electron due to this field is radially inward with magnitude $eE_+(d)$. Therefore, magnitude of the net force on the electron is

$$|\vec{F}| = |\vec{F}_e| - |\vec{F}_+| = \frac{e^2}{16\pi\epsilon_0 d^2} - \frac{e^2 d}{2\pi\epsilon_0 R^3}, \quad |\vec{F}| = 0 \rightarrow d = \frac{R}{2}.$$

(c) Since the net charge inside the sphere is zero, by Gauss's law, total electric flux through the outer surface of the sphere will be zero.

(d) Magnitude of the electric field created by the positive charge on the circle is

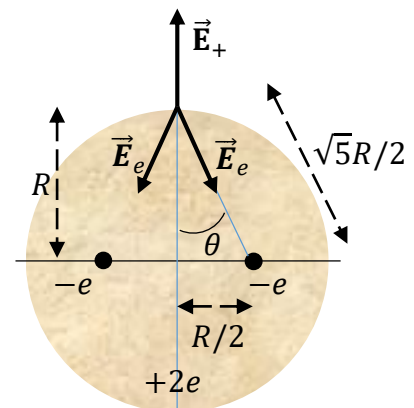
$$E_+ = \frac{e}{2\pi\epsilon_0 R^2}$$

Magnitude of the electric field created by the electron at the same point is

$$E_e = \frac{e}{5\pi\epsilon_0 R^2}$$

From the figure, we see that $\cos \theta = 2\sqrt{5}/5$, and the magnitude of the electric field on the equatorial circle of the sphere is

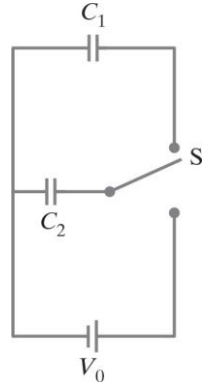
$$E = E_+ - 2E_e \cos \theta \rightarrow E = \frac{e}{2\pi\epsilon_0 R^2} - 2 \frac{e}{5\pi\epsilon_0 R^2} \frac{2\sqrt{5}}{5} \rightarrow E = \left(\frac{50 - 16\sqrt{5}}{25} \right) \frac{e}{4\pi\epsilon_0 R^2}$$



3. The switch S in the figure is initially connected downward so that capacitor C_2 becomes fully charged by the battery of voltage V_0 .

(a) (15 Pts.) If the switch is then connected upward, determine the charge on each capacitor after the switching.

(b) (15 Pts.) What is the change in the energy stored in the capacitors because of the switching?



Solution: (a) Initially, the charge on C_2 is $Q_0 = C_2 V_0$. When the switch is connected upward, some of this charge will flow to C_1 until the voltage on both capacitors are same. Let Q'_1 and Q'_2 denote charges on capacitors C_1 and C_2 after the switch is connected upward. Since the charge is conserved, we have

$$Q'_1 + Q'_2 = Q_0, \quad \frac{Q'_1}{C_1} = \frac{Q'_2}{C_2} \rightarrow Q'_1 = \frac{C_1 Q_0}{C_1 + C_2} = \frac{C_1 C_2 V_0}{C_1 + C_2}, \quad Q'_2 = \frac{C_2 Q_0}{C_1 + C_2} = \frac{C_2^2 V_0}{C_1 + C_2}.$$

(b) Before the switch is connected upward the initial energy stored in C_2 is

$$E_i = \frac{1}{2} C_2 V_0^2.$$

After the switch is closed upward, the final total energy of the system becomes

$$E_f = \frac{Q_1'^2}{2 C_1} + \frac{Q_2'^2}{2 C_2} \rightarrow E_f = \frac{C_2^2 V_0^2}{2(C_1 + C_2)}.$$

Therefore,

$$\Delta E = E_f - E_i = \frac{C_2^2 V_0^2}{2(C_1 + C_2)} - \frac{1}{2} C_2 V_0^2 \rightarrow \Delta E = \frac{-C_1 C_2 V_0^2}{2(C_1 + C_2)} = \left(\frac{-C_1}{C_1 + C_2} \right) E_i.$$